Correlation coefficient

The measure of linear association between two variables $X$ and $Y$

$$\gamma = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}, \quad -1 \leq \gamma \leq 1$$

where

$$S_{xy} = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

$$S_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$S_{yy} = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

$\gamma^2$ : Sample coefficient of determination

- Represent the proportion of the variation of $S_{yy}$ explained by the regression of $Y$ on $X$.
- A $\gamma$ of 0.9 means that 0.81(81%) of the total variation of $Y$ in our sample can be explained by a linear relationship with values of $X$. 
\( \gamma = 0 \) implies a lack of linearity, but not a lack of association.
\[ f = w \]

(\text{gray - levels })
\[ f \approx w \]
$f \neq w$
Matching by correlation

Correlation coefficients for 2D images

Given two images \( f(x,y) \) and \( w(l,m) \) \(-k \leq l, m \leq k\) and \( n=2k+1 \)

Let \( \bar{f} \) and \( \bar{w} \) be the mean gray levels of \( f(x,y) \) and \( w(l,m) \)

\[
\bar{f} = \frac{1}{n^2} \sum_{i=-k}^{k} \sum_{m=-k}^{k} f(x+i, y+m)
\]

\[
\bar{w} = \frac{1}{n^2} \sum_{l=-k}^{k} \sum_{m=-k}^{k} w(l,m)
\]

\[
\gamma(x,y) = \frac{\sum_{l} \sum_{m} [f(x+l, y+m) - \bar{f}] \cdot [w(l,m) - \bar{w}]}{\sqrt{\left(\sum_{l} \sum_{m} [f(x+l, y+m) - \bar{f}]^2 \cdot \sum_{l} \sum_{m} [w(l,m) - \bar{w}]^2\right)}}
\]
\[
\gamma (x, y) = \frac{\sum \sum [f(x+l, y+m) \cdot w(l, m)] - n^2 \bar{f} \cdot \bar{w}}{\left[ \sum \sum f^2(x+l, y+m) - n^2 \bar{f}^2 \right]^{\frac{1}{2}} \left[ \sum \sum w^2(l, m) - n^2 \bar{w}^2 \right]^{\frac{1}{2}}}
\]

Normalized measure: let \( \bar{w} = 0 \)

\[
\gamma_N (x, y) = \frac{\sum \sum [f(x+l, y+m) \cdot w(l, m)]}{\left[ \sum \sum f^2(x+l, y+m) - n^2 \bar{f}^2 \right]^{\frac{1}{2}} \left[ \sum \sum w^2(l, m) \right]^{\frac{1}{2}}}
\]

- Very sensitive to noise, poor detection
- Good localization
Cosine measure: Let $\overline{f} = 0$

Consider the two image $f$ and $w$ as two vectors $\overline{F}$ and $\overline{W}$, then

$$\cos \theta = \frac{\overline{F} \cdot \overline{W}}{|\overline{F}| |\overline{W}|}$$

$$\gamma_0 (x, y) = \frac{\sum_{l} \sum_{m} [f(x+l, y+m) \cdot w(l, m)]}{\left[ \sum_{l} \sum_{m} f^2(x+l, y+m) \cdot \sum_{l} \sum_{m} w^2(l, m) \right]^{\frac{1}{2}}}$$

- Less sensitive to noise
- Mean localization
Projection measure: let \( \sum \sum f^2(x + l, y + m) = 1 \)

\[
\gamma_p(x, y) = \frac{\sum \sum [f(x + l, y + m) \cdot w(l, m)]}{\left[\sum \sum w^2(l, m)\right]^{1/2}}
\]

- Least sensitive to noise, very good detection
- Poor localization

Convolution measure (cross correlation):

Let \( \sum \sum w^2(l, m) = 1 \)

\[
\gamma_c = \sum \sum f(x + l, y + m) \cdot w(l, m)
\]
Subpixel matching

Interpolation around the peak of $\gamma(x,y)$ using
(second-degree polynomial) curve fitting

Let $\gamma(x,y) = \max\{\gamma(i,j) | (i,j) \in N(x,y)\}$

The subpixel correction term $(\triangle x, \triangle y)$:

$$\triangle x = \frac{1}{2} \frac{\gamma(x-1,y) - \gamma(x+1,y)}{\gamma(x+1,y) + \gamma(x-1,y) - 2\gamma(x,y)}$$

$$\triangle y = \frac{1}{2} \frac{\gamma(x,y-1) - \gamma(x,y+1)}{\gamma(x,y+1) + \gamma(x,y-1) - 2\gamma(x,y)}$$
Effects of smoothing
Correlation coefficient for color images

令 $\bar{C}_M(i, j)$ 爲標準影像在像素點 $(i, j)$ 的 RGB 向量，$\bar{C}_M(i, j) = (R(i, j), G(i, j), B(i, j))$

$\bar{C}_S(x, y)$ 爲測試影像在像素點 $(x, y)$ 的 RGB 向量，$\bar{C}_S(x, y) = (\hat{R}(x, y), \hat{G}(x, y), \hat{B}(x, y))$

則定義標準圖形與待測圖形之彩色影像點相關係數 $(\gamma_I)$ 爲：

$$\gamma_I = \frac{\sum_{i,j} \bar{C}_M(i, j) \cdot \bar{C}_S(x+i, y+j) - 3N_w \times \mu_I \times \hat{\mu_I}}{\sqrt{\left(\sum_{i,j} T_I(i, j) - 3N_w \times \mu_I^2\right) \times \left(\sum_{i,j} \hat{T}_I(x+i, y+j) - 3N_w \times \hat{\mu_I}^2\right)}}$$

其中 $T_I(i, j) = [R^2(i, j) + G^2(i, j) + B^2(i, j)]$

$\hat{T}_I(x+i, y+j) = [\hat{R}^2(x+i, y+j) + \hat{G}^2(x+i, y+j) + \hat{B}^2(x+i, y+j)]$

$$\mu_I = \frac{1}{3 \times N_w^2} \sum_{j=-W}^{W} \sum_{i=-W}^{W} [R(i, j) + G(i, j) + B(i, j)]$$

$$\hat{\mu}_I = \frac{1}{3 \times N_w^2} \sum_{j=-W}^{W} \sum_{i=-W}^{W} [\hat{R}(x+i, y+j) + \hat{G}(x+i, y+j) + \hat{B}(x+i, y+j)]$$
<table>
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<th>彩色影像點比對法</th>
<th>灰階影像點比對法</th>
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