Correlation coefficient

The measure of linear association between two variables X and Y

$$\gamma = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}, \quad -1 \le \gamma \le 1$$

where

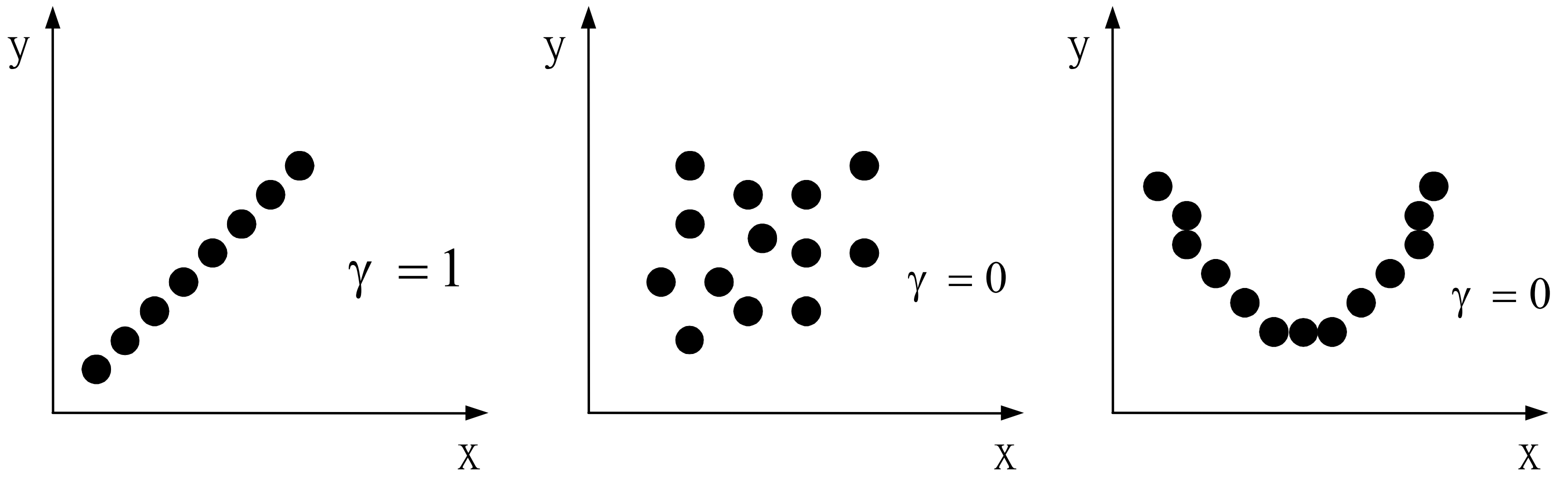
$$S_{xy} = \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

$$S_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$S_{yy} = \sum_{i=1}^{n}(y_i - \bar{y})^2$$

γ^2 : Sample coefficient of determination

- ⇨ Represent the proportion of the variation of S_{yy} explained by the regression of Y on X.
- ⇨ A γ of 0.9 means that 0.81(81%) of the total variation of Y in our sample can be explained by a linear relationship with values of X.



$\gamma = 0$ implies a lack of linearity, but not a lack of association.

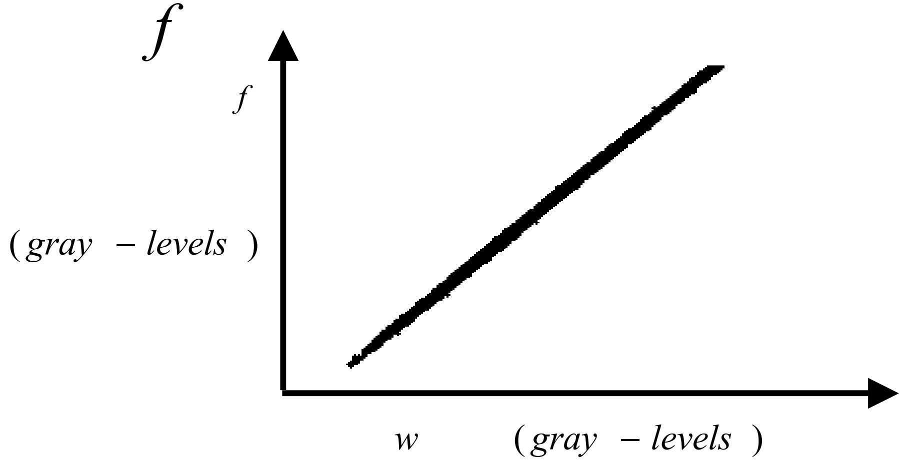
$$f = w$$

MPX 2
EN5025A
6C235P2 705F
C26 C27

f

MPX 2
EN5025A
6C235P2 705F
C26 C27

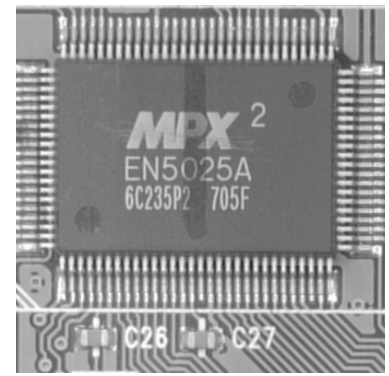
w



f
(gray – levels)
w
(gray – levels)

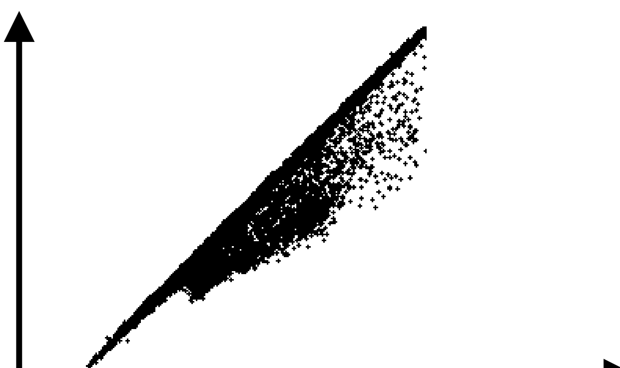
$f \approx w$

f



w

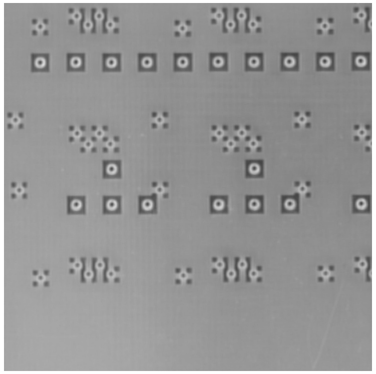
f (gray – levels)



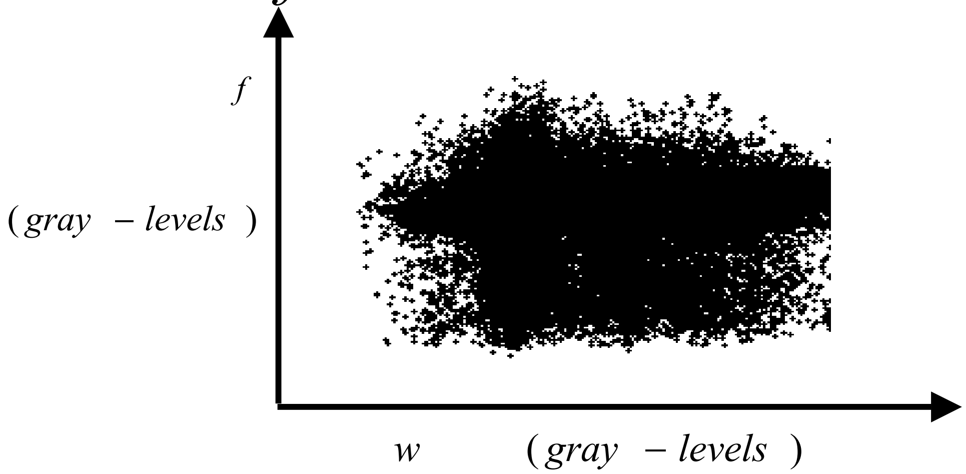
w (gray – levels)

$$f \neq w$$

MPX 2
EN5025A
6C235P2 705F
C26
C27



f w



f (gray − levels)
w (gray − levels)

Matching by correlation

Correlation coefficients for 2D images

Given two images *f(x,y)* and *w(l,m)* $-k \leq l,\ m \leq k$ *and* $n=2k+1$

Let $\bar{f}$ and $\bar{w}$ be the mean gray levels of *f(x,y)* and *w(l,m)*

$$\bar{f} = \frac{1}{n^2} \sum_{l-k}^{k} \sum_{m-k}^{k} f(x+l, y+m)$$

$$\bar{w} = \frac{1}{n^2} \sum_{l-k}^{k} \sum_{m-k}^{k} w(l, m)$$

$$\gamma(x,y) = \frac{\sum_{l} \sum_{m} \left[f(x+l, y+m) - \bar{f} \right] \cdot \left[w(l,m) - \bar{w} \right]}{\left\{ \sum_{l} \sum_{m} \left[f(x+l, y+m) - \bar{f} \right]^2 \cdot \sum_{l} \sum_{m} \left[w(l,m) - \bar{w} \right]^2 \right\}^{1/2}}$$

$$\gamma(x,y) = \frac{\sum_l \sum_m \left[f(x+l, y+m) \cdot w(l,m) \right] - n^2 \overline{f} \cdot \overline{w}}{\left[\left(\sum \sum f^2(x+l, y+m) - n^2 \overline{f}^2 \right) \left(\sum \sum w^2(l,m) - n^2 \overline{w}^2 \right) \right]^{1/2}}$$

Normalized measure: let $\overline{w}$=0

$$\gamma_N(x,y) = \frac{\sum_l \sum_m \left[f(x+l, y+m) \cdot w(l,m) \right]}{\left[\left(\sum \sum f^2(x+l, y+m) - n^2 \overline{f}^2 \right) \left(\sum \sum w^2(l,m) \right) \right]^{1/2}}$$

- Very sensitive to noise, poor detection
- Good localization

Cosine measure: Let $\overline{f} = 0$

Consider the two image f and w as two vectors $\vec{F}$ and $\vec{W}$, then

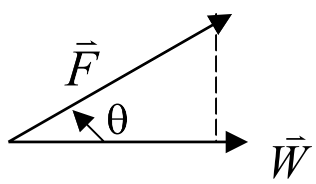
$$\cos\theta = \frac{\vec{F} \cdot \vec{W}}{|\vec{F}||\vec{W}|}$$

$$\gamma_\theta(x, y) = \frac{\sum_l \sum_m [f(x+l, y+m) \cdot w(l, m)]}{\left[\sum_l \sum_m f^2(x+l, y+m) \cdot \sum_l \sum_m w^2(l, m)\right]^{1/2}}$$

- Less sensitive to noise
- Mean localization

Projection measure: let $\sum\sum f^2(x+l, y+m) = 1$

$$\gamma_p(x,y) = \frac{\sum_l \sum_m \left[f(x+l, y+m) \cdot w(l,m)\right]}{\left[\sum_l \sum_m w^2(l,m)\right]^{1/2}}$$



- Least sensitive to noise, very good detection
- Poor localization

Convolution measure (cross correlation):

Let $\sum\sum w^2(l,m) = 1$

$$\gamma_c = \sum_l \sum_m f(x+l, y+m) \cdot w(l,m)$$

Subpixel matching

Interpolation around the peak of $\gamma(x, y)$ using

(second-degree polynomial) curve fitting

Let $\gamma(x, y) = \max\{\gamma(i, j) \mid (i, j) \in N(x, y)\}$

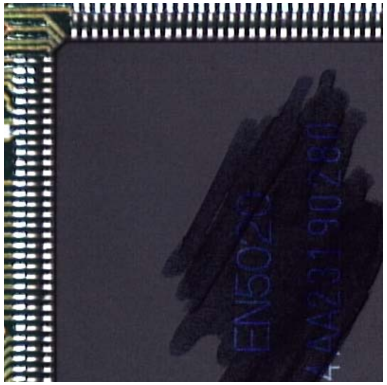
The subpixel correction term (△x, △y):

$$\triangle x = \frac{1}{2} \frac{\gamma(x-1, y) - \gamma(x+1, y)}{\gamma(x+1, y) + \gamma(x-1, y) - 2\gamma(x, y)}$$

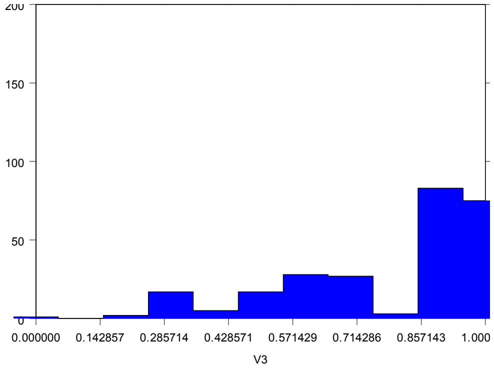
$$\triangle y = \frac{1}{2} \frac{\gamma(x, y-1) - \gamma(x, y+1)}{\gamma(x, y+1) + \gamma(x, y-1) - 2\gamma(x, y)}$$

Effects of smoothing

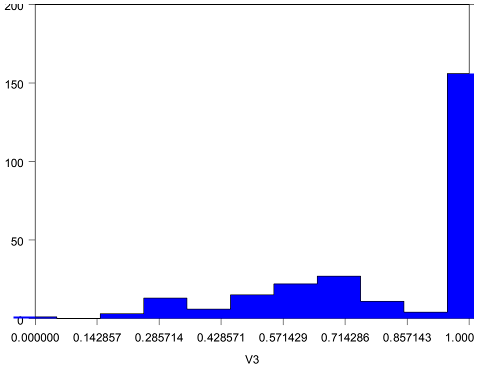
標準影像



待測影像



無smoothing



有smoothing

Correlation coefficient for color images

令 $\vec{C}_M(i,j)$ 爲標準影像在像素點*(i,j)* 的RGB向量，$\vec{C}_M(i,j)=(R(i,j),G(i,j),B(i,j))$

$\vec{C}_S(x,y)$ 爲測試影像在像素點*(x,y)*的RGB向量，$\vec{C}_S(x,y)=(\hat{R}(x,y),\hat{G}(x,y),\hat{B}(x,y))$

則定義標準圖形與待測圖形之彩色影像點相關係數（γ_I）爲：

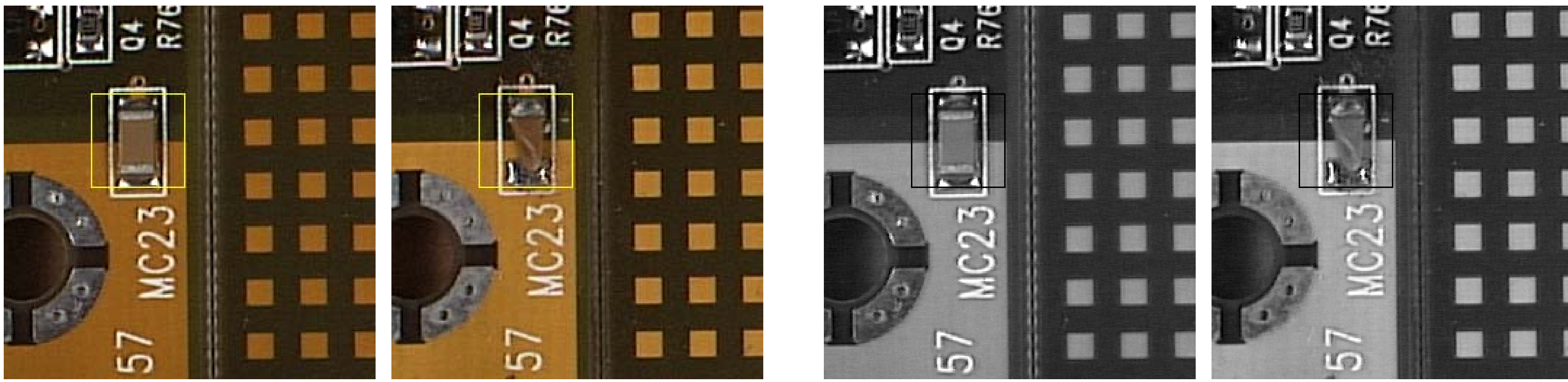
$$\gamma_I=\frac{\sum_i\sum_j \vec{C}_M(i,j)\cdot\vec{C}_S(x+i,y+j)-3N_W\times\mu_I\times\hat{\mu}_I}{\sqrt{\left(\sum_i\sum_j T_I(i,j)-3N_W\times\mu_I{}^2\right)\times\left(\sum_i\sum_j \hat{T}_I(x+i,y+j)-3N_W\times\hat{\mu}_I{}^2\right)}}$$

其中 $T_I(i,j)=[R^2(i,j)+G^2(i,j)+B^2(i,j)]$

$$\hat{T}_I(x+i,y+j)=[\hat{R}^2(x+i,y+j)+\hat{G}^2(x+i,y+j)+\hat{B}^2(x+i,y+j)]$$

$$\mu_I=\frac{1}{3\times N_W}\sum_{j=-v}^{v}\sum_{i=-W}^{W}[R(i,j)+G(i,j)+B(i,j)]$$

$$\hat{\mu}_I=\frac{1}{3\times N_W}\sum_{j=-v}^{v}\sum_{i=-W}^{W}[\hat{R}(x+i,y+j)+\hat{G}(x+i,y+j)+\hat{B}(x+i,y+j)]$$



(a) (b) (c) (d)

	彩色影像點比對法		灰階影像點比對法	
	正常影像	瑕疵影像	正常影像	瑕疵影像
	最大之相關係數值			
影像圖	圖(a)	圖(b)	圖(c)	圖(d)
平均值	0.936515	0.697862	0.916716	0.640878
標準差	0.001191	0.001679	0.001358	0.001877