Corner detection in binary images

- A corner point is an edge point
- A corner point occurs when the edge direction changes

An ideal corner detector :

- Detection
- Localization
- Stability

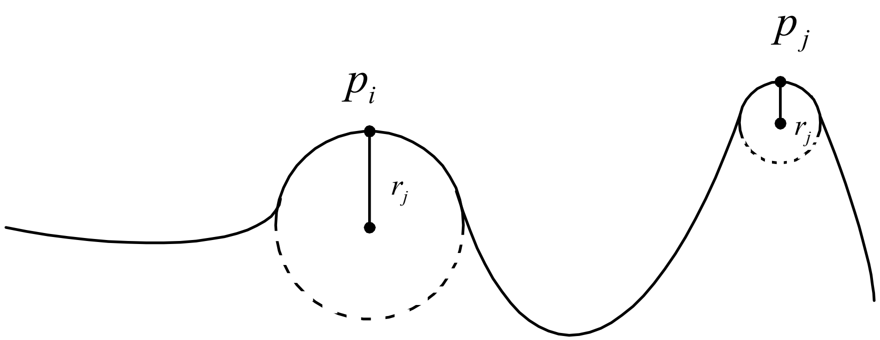
Measures of cornerity

- Curvature

Let $P = \{x(p), y(p)\}$

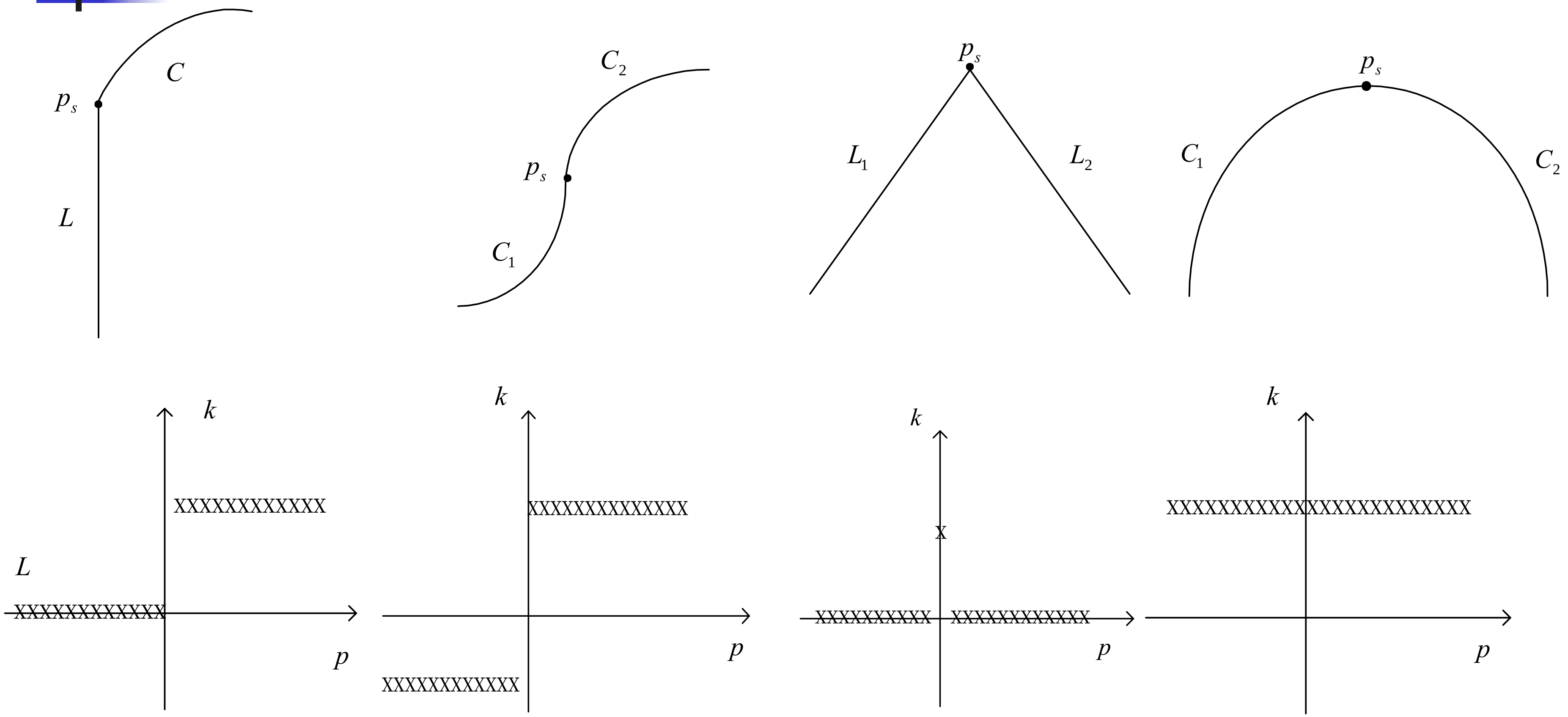
Tangent : $t(p) = \frac{dy}{dx}$

Curvature: $\kappa(p) = \frac{dt(p)}{dp}$



- Radius

$$\kappa(p) = \frac{1}{r(p)}$$



C
p_s
L
C_2
p_s
C_1
p_s
L_1
L_2
p_s
C_1
C_2
k
p
L
k
p
k
p
k
p

Let P be a set of digital boundary

$$P = \{p_i = (x_i, y_i), i = 1, 2, \ldots, n\}$$

Bennett - MacDonald method

Tangent angle:

$$\varphi_i = \tan^{-1}\left[\frac{y_i - y_{i-1}}{x_i - x_{i-1}}\right]$$

Curvature:

$$\kappa_i = \varphi_i - \varphi_{i-1}$$

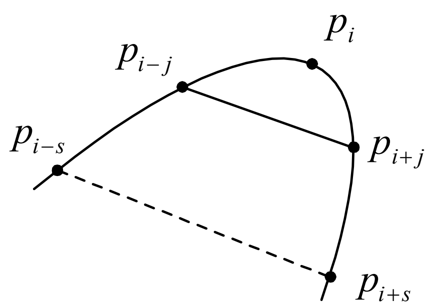
Improved B-M method with smoothing

Tangent angle

$$\bar{\varphi}_i = \tan^{-1}\left[\frac{1}{s}\sum_{j=1}^{s}\left(\frac{Y_{i+j} - Y_{i-j}}{X_{i+j} - X_{i-j}}\right)\right]$$

$$\bar{\kappa}_i = \frac{1}{s}\sum_{j=1}^{s}\left(\bar{\varphi}_{i+j} - \bar{\varphi}_{i-j}\right)$$

where s = the region of support



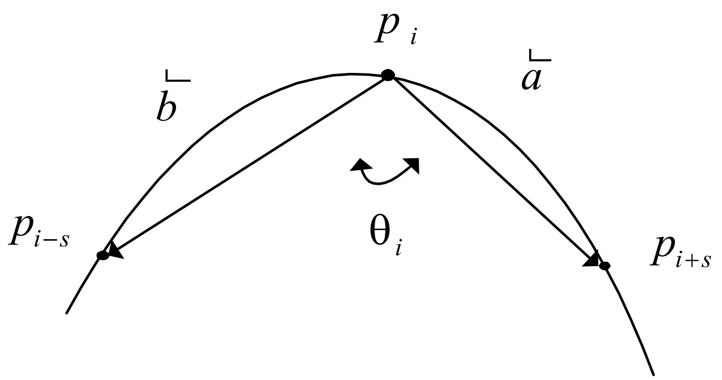
Rosenfeld-Johnson method

Let $\vec{a}_{is} = (x_i - x_{i+s}, y_i - y_{i+s})$

$\vec{b}_{is} = (x_i - x_{i-s}, y_i - y_{i-s})$

The angle between vectors $\vec{a}_{is}$ *and* $\vec{b}_{is}$

$$\cos\theta_i = \frac{\vec{a}_{is} \cdot \vec{b}_{is}}{|\vec{a}_{is}| \cdot |\vec{b}_{is}|}$$



$|\cos\theta_i| \approx 1$, p_i is on a line

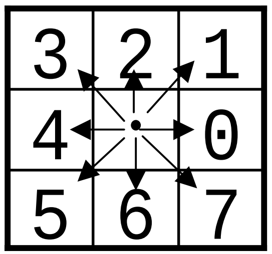
$|\cos\theta_i| << 1$, p_i is a corner

Freeman – Davis method

Define an octal chain code $\{a_i\ , i = 1,2,\cdots,n\}$

where $a_i \in \{0,1,2,3,4,5,6,7\}$

let a_{ix} and a_{iy} be the x and y components of a_i



a_i	a_{ix}	a_{iy}
0	1	0
1	1	1
2	0	1
3	-1	1
4	-1	0
5	-1	-1
6	0	-1
7	1	-1

let L_j^s = a line segment spanning s chain links

$$= a_{j-s+1} \cdot a_{j-s+2} \ \ldots \ a_j$$

The x and y components of L_j^s

$$X_j^s = \sum_{i=j-s+1}^{j} a_{ix}$$

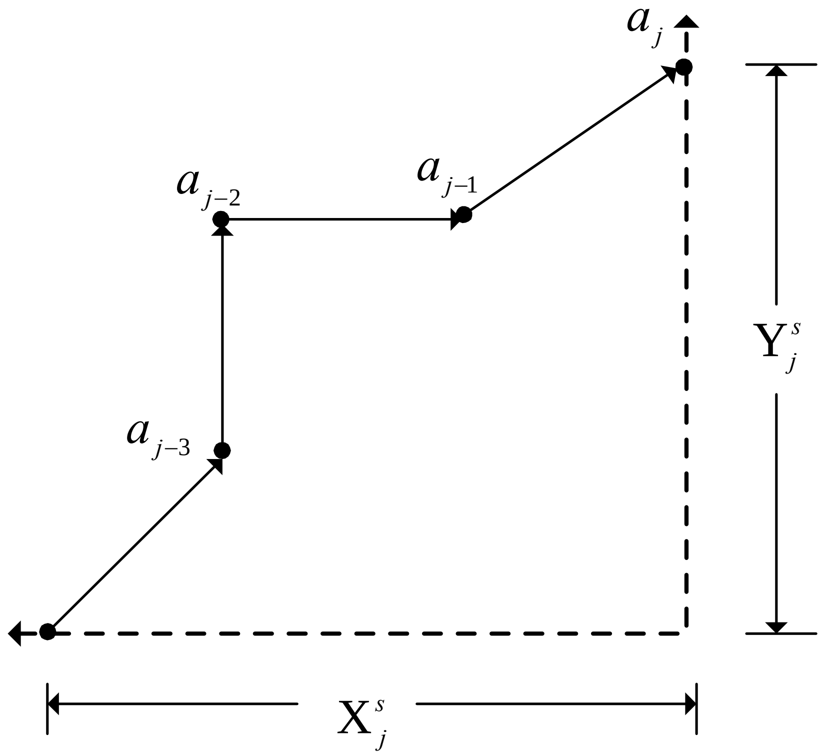
$$Y_j^s = \sum_{i=j-s+1}^{j} a_{iy}$$

The tangent angle

$$\theta_j = \tan^{-1}\left(\frac{Y_j^s}{X_j^s}\right)$$

The curvature

$$\kappa_j = \theta_{j+1} - \theta_{j-1}$$



Define the cornerity at p_j

$$C_j = \ln(t_1) \cdot \left(\sum_{i=j}^{j+s} \kappa_i \right) \cdot \ln(t_2)$$

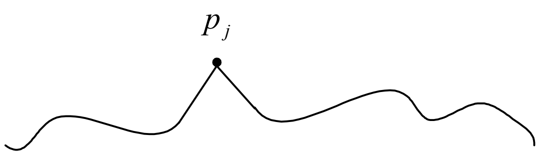
where
$$t_1 = \max\left\{\, t \mid \kappa_{j-v} \in (-\Delta, \Delta), \forall v, 1 \le v \le t \right\}$$

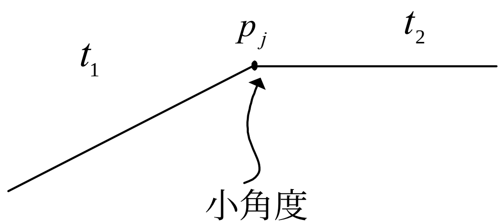
$$t_2 = \max\left\{\, t \mid \kappa_{j+s+v} \in (-\Delta, \Delta), \forall v, 1 \le v \le t \right\}$$

$$\Delta = \tan^{-1}\left(\frac{1}{s-1} \right)$$

s is the region of support

t_1 and t_2 represent straight-line segments on both sides of a corner.





Least – squares curve fitting

Given a segment of $2s+1$ boundary points

$$N(p_i) = \{p_j = (x_j, y_j), j = i-s, i-s+1, \ldots, i, i+1, \ldots, i+s\}$$

$$\text{Min} \sum_{i=i-s}^{i+s} [y_i - f(x_i)]^2$$

where

$$f(x_i) = a + bx_i + cx_i^2 + dx_i^3, \forall\, i$$

The cubic polynomial curve $f(x)$ is estimated, in the least-square sense, by

$$A \cdot X = b$$

where

$$A = \begin{bmatrix} \sum 1 & \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & \sum x_i^5 \\ \sum x_i^3 & \sum x_i^4 & \sum x_i^5 & \sum x_i^6 \end{bmatrix}$$

$$X = [a, b, c, d]$$

$$b = \left[\sum y_i, \sum y_i x_i, \sum y_i x_i^2, \sum y_i x_i^3\right]^T$$

and

$$X = A^{-1} \cdot b$$

For curve: $y = a + bx + cx^2 + dx^3,$

The tangent slope $y' = f'(x_i)$

The curvature $\kappa_i = \frac{y''}{\left[1 + (y')^2\right]^{3/2}}$

Eigenvalues of covariance matrices

Given a segment of $2s+1$ boundary points

$$N(p_i) = \{p_j = (x_j, y_j), j = i-s, i-s+1, \ldots, i, i+1, \ldots, i+s\}$$

The covariance matrix M of the segment

$$M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

where

$$m_{11} = \left[\frac{1}{2s+1}\sum_{j=i-s}^{i+s} x_j^{\ 2}\right] - \overline{m}_x^{\ 2}$$

$$\overline{m}_x = \frac{1}{2s+1}\sum_{j=i-s}^{i+s} x_j$$

$$m_{12} = m_{21} = \left[\frac{1}{2s+1}\sum_{j=i-s}^{i+s} x_j \cdot y_j\right] - \overline{m}_x^{\ 2} \cdot \overline{m}_y^{\ 2}$$

$$\overline{m}_y = \frac{1}{2s+1}\sum_{j=i-s}^{i+s} y_j$$

$$m_{22} = \left[\frac{1}{2s+1}\sum_{j=i-s}^{i+s} y_j^{\ 2}\right] - \overline{m}_y^{\ 2}$$

The two eigenvalues of the matrix M

$$\lambda_1 = \frac{1}{2}\left(m_{11} + m_{22} + \left[(m_{11} - m_{22})^2 + 4m_{12}^2 \right]^{\frac{1}{2}} \right)$$

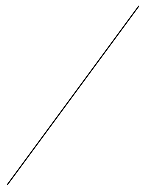
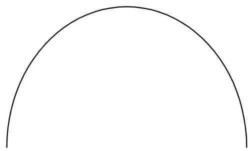
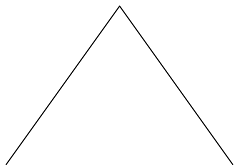
$$\lambda_2 = \frac{1}{2}\left(m_{11} + m_{22} - \left[(m_{11} - m_{22})^2 + 4m_{12}^2 \right]^{\frac{1}{2}} \right)$$

IF $N(p_i)$ is an ellipse,

$\sqrt{\lambda_1}$ and $\sqrt{\lambda_2}$ are the semimajor and semiminor axial length of the ellipse

$\lambda_2 \approx 0$ for a line

$\lambda_2 >> 0$ for a corner.

Curve			λ_2 *value* (21 data points)
	Line (slope angle)	$0^{\square}$ $20^{\square}$ $40^{\square}$ $60^{\square}$ $80^{\square}$	-1.9×10^{-6} 3.8×10^{-6} -7.6×10^{-6} 3.8×10^{-6} -2.6×10^{-6}
	Circle (Radius in pixel)	30 50 70 90	0.256 0.090 0.043 0.024
	Included angle	$30^{\square}$ $50^{\square}$ $70^{\square}$ $90^{\square}$	6.969 5.952 4.845 3.673

Wavelet decomposition approach

$$Let\ T = \{\theta(i), i = 1,2,\ldots,n\},$$

$$\text{where}\ \theta_i = \text{tangent angle of point}\ p_i$$

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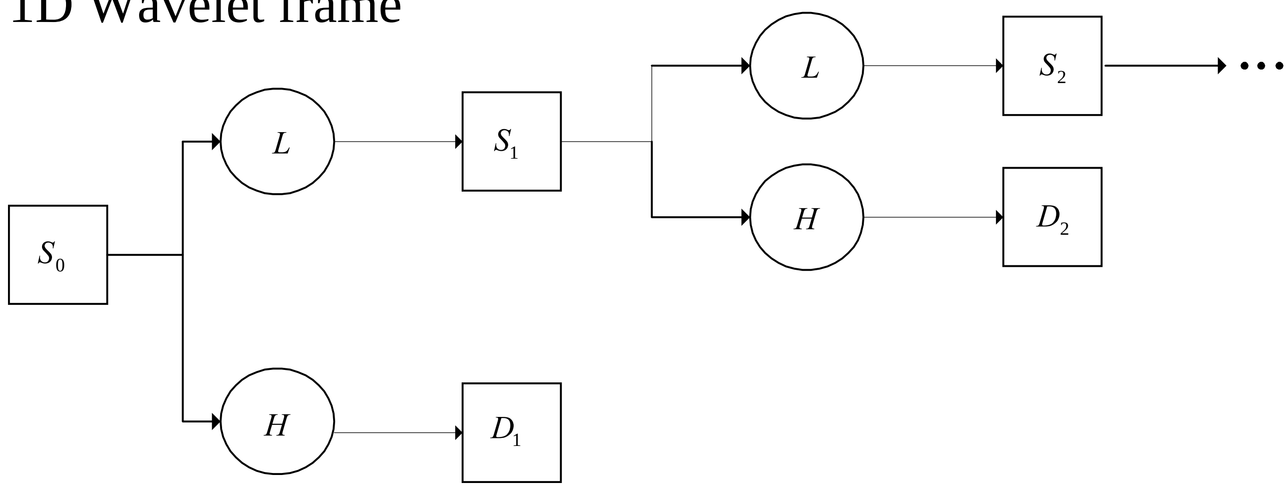
1D Wavelet frame



$$Let\ S_0(i) = \theta(i), i = 1,2,\ldots,n$$

$$S_1(x) = \frac{1}{I}\sum_{i=0}^{I-1} L(i)\cdot S_0(x+i)$$

$$D_1(x) = \frac{1}{J}\sum_{j=0}^{i-1} H(j)\cdot S_0(x+j)$$

$$\vdots$$

$$S_N(x) = \frac{1}{I}\sum L(i)\cdot S_{N-1}(x+i)$$

$$D_N(x) = \frac{1}{J}\sum H(j)\cdot S_{N-1}(x+j)$$

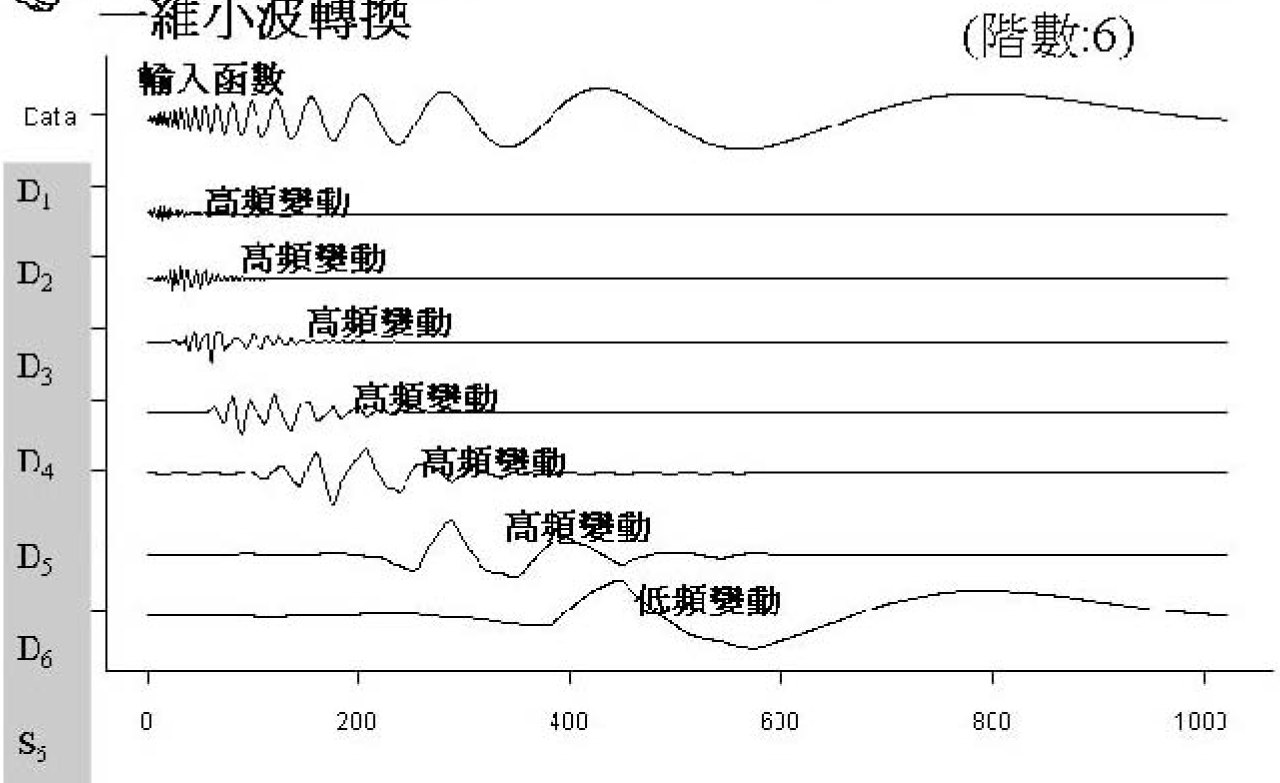
where $L(i)$ = a low-pass filter of length I

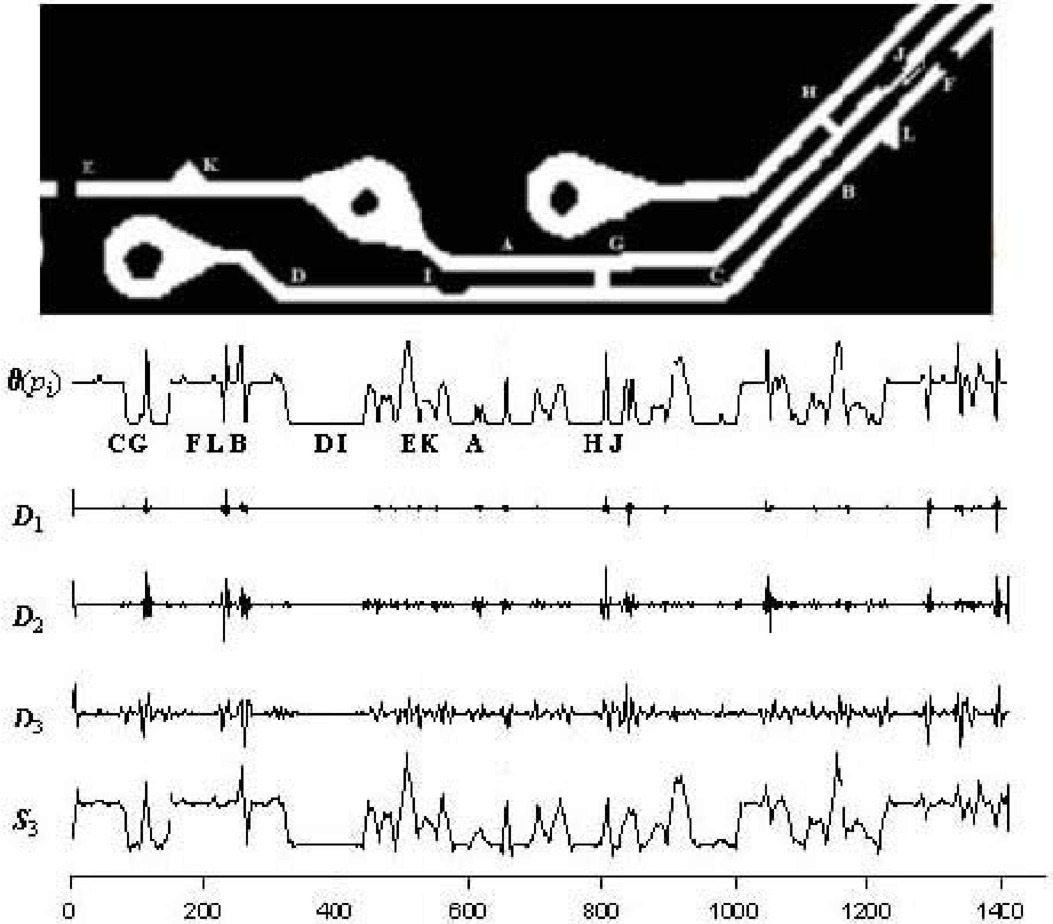
$H(j)$ = a low-pass filter of length J

Wavelet basis	*L*	*H*
Haar	[0] 0.707106781 [1] 0.707106781	[0] 0.707106781 [1]-0.707106781
D4 (Daubechies)	[0] 0.482962913 [1] 0.836516303 [2] 0.224143868 [3]-0.129409522	[0] 0.129409522 [1] 0.224143868 [2]-0.836516303 [3] 0.482962913

小波轉換簡介

☞ 一維小波轉換




E K
D I A G C
H J F L B
θ(p_i)
C G F L B D I E K A H J
D_1
D_2
D_3
S_3
0 200 400 600 800 1000 1200 1400